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Scaling properties of $S > \frac{1}{2}$ Ising spin glasses

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Abstract. Transfer-matrix-based calculations of the defect energy in two-dimensional Ising spin glasses yield a $T = 0$ exponent y which depends on the spin quantum number S. For a sufficiently large S there is a possibility of freezing at finite T in two dimensions. The Emery–Swendsen version of Migdal–Kadanoff recursion formula is used to investigate the $S = 1$ two- and three-dimensional spin glasses at finite temperatures. The phase diagram on the T–single-ion anisotropy plane allows for no re-entrancy predicted by the mean-field approach.

1. Introduction

Years of research on frustrated spin systems (see, e.g. Binder and Young 1986) have led to a basic understanding of the equilibrium properties of the $S = \frac{1}{2}$ Ising and of the classical Heisenberg spin glasses. Much less is known, however, about quantum frustrated systems and about the Ising systems with a general spin quantum number S. Experimentally studied spin glasses often consist of $S > \frac{1}{2}$ spins and such systems may exhibit Ising-like anisotropy.

In this paper, we focus on the $T = 0$ scaling properties of the $S > \frac{1}{2}$ systems described by the Hamiltonian

$$H = -\sum_{(ij)} J_{ij} S_i S_j + \sum_i \Delta_i \, S_i^2 \tag{1}$$

where the spin S_i takes on values $-S, \; -S+1, \ldots, S-1, S$ and the exchange couplings involve nearest neighbours. We consider the case of a Gaussian spin glass (GSG), when the couplings are Gaussian distributed with a zero mean and a dispersion J_0, and of a bimodal spin glass (BSG), when the probability to find a coupling J_{ij} is given by $P(J_{ij}) = [\delta(J_{ij} - J_0) + \delta(J_{ij} + J_0)]/2$. In (1), Δ_i denotes the local anisotropy constant. In most of the paper we consider anisotropy to be uniform $\Delta_i = \Delta$.

The $S = 1$ model has already been studied within the mean-field theory (Ghatak and Sherrington 1977, Lage and de Almeida 1982, Mottishaw and Sherrington 1985). In particular, it has been found that for some range of Δ the paramagnet–spin-glass transition becomes first order and re-entrancy phenomena take place.

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The questions we ask in this paper are as follows: (a) what is the phase diagram corresponding to (1), (b) what is the effect of the spin quantum number S on the lower critical dimensionality of the system.

We seek answers to these questions by determining the $T = 0$ scaling properties of the two-dimensional (2D) Ising systems with $S \leqslant 3$ and by calculating effective coupling constants within the Migdal–Kadanoff renormalization group (MKRG) scheme for two- and three-dimensional $S = 1$ systems. The $T = 0$ scaling results are based on the numerically exact transfer-matrix method. The MKRG scheme, on the other hand, is approximate but it allows one to study much larger length scales and to consider essentially arbitrary space dimensionalities.

The basic concept of the $T = 0$ scaling theory (Banavar and Cieplak 1982, Bray and Moore 1984, McMillan 1984, Bray and Moore 1987, Bray 1988) is that of a scaling stiffness or a scale-dependent coupling energy, $\delta E(L)$. This coupling is determined by studying the sensitivity to boundary conditions of the ground-state energy of finite blocks of length L. $\delta E(L)$ is a characteristic measure of that sensitivity; it is defined in section 2. In the MKRG scheme (Migdal 1976, Kadanoff 1976, Southern and Young 1977), on the other hand, one studies moments of the probability distribution of the exchange couplings characterizing systems at length scale L. In the spin-glass phase, the second moment, or the dispersion, $\sigma_J(L)$, has the dominating L-dependence and serves as a characteristic measure of the coupling strength.

In the ordered phase at $T = 0$ both $\delta E(L)$ and $\sigma_J(L)$ scale as

$$\delta E(L) \qquad \sigma_J(L) \sim L^y . \tag{2}$$

For systems below the lower critical dimensionality (LCD), y is negative and a phase transition occurs at $T = 0$. Above the LCD, on the other hand, y is positive and the transition occurs at a non-zero T_c. The marginal case, $y = 0$, corresponds to the system being at its LCD.

In this paper, we study the properties of y as a function of S. The summary of our transfer-matrix-based findings for $D = 2$ and $\Delta = 0$ is shown in figure 1. We see that tendencies towards ordering generally increase with S, i.e. the effects

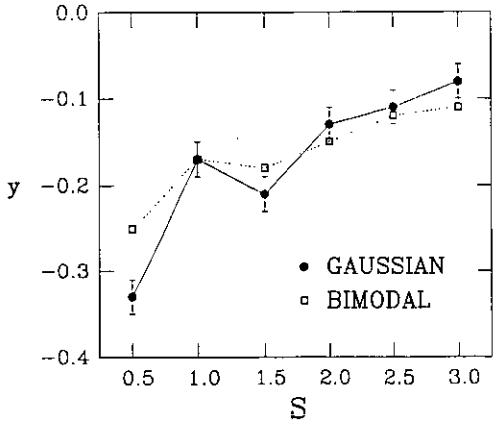

Figure 1. The $T = 0$ exponent y obtained by the transfer matrix-method for two-dimensional spin glasses with the spin quantum number S. The anisotropy parameter Δ is set equal to zero. The error bars for systems with the bimodal couplings are comparable to those with the Gaussian couplings.

of frustration get weaker. The lines in figure 1 are merely guides to the eye. In fact, the results for the Gaussian system with $S \leqslant 3$ can also be represented by a superposition of two approximately straight lines: one for the half-integer S and the other for the integer S. For $S = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ we get $y = -0.33$, -0.21, and -0.11 respectively. For $S = 1$, 2, and 3 our results are correspondingly -0.17, -0.13, and -0.08. The error bars due to statistics are of order of 0.03. If the extrapolation of these straight lines to larger values of S is justified (the alternative is that a saturation at a negative value of y is reached) then there is a critical S_c above which there is a spin-glass ordering in two dimensions. In the half-integer case the ordering would then be predicted to arise for $S = \frac{9}{2}$, and in the integer case for $S = 5$, which is out of the range found in solid-state systems. However, our results point to possible integer–half-integer effects that could also be found in three-dimensional spin glasses.

We also observe that S of 1 seems to be special: the bimodal and Gaussian distributions give rise to the same y of -0.17. Notice that in the case of 2D 3-state Potts spin glasses $y = -0.61$ and -0.44 for the Gaussian and bimodal couplings respectively (Banavar and Cieplak 1989). The S dependence of the exponent y is also demonstrated by the Migdal–Kadanoff approach.

We start describing our results in section 2 by giving details of our transfer matrix based calculations of $\delta E(L)$. In section 3 we generalize the Emery–Swendsen (Emery and Swendsen 1977a, 1977b) version of the MKRG scheme for $S > \frac{1}{2}$ to random $S = 1$ systems and present results for the 2D and 3D spin glasses. In particular, we obtain the phase diagram on the T–Δ plane for the GSG and BSG in $D = 3$.

Our calculations have dealt with the Ising systems but they suggest that the $T = 0$ scaling exponent y for quantum Heisenberg spin glasses may depend on the spin quantum number S. This is in contrast to what happens in unfrustrated systems where there is no S dependence in y. It would be interesting to find out whether experimental results suggest any S dependence of the exponents.

2. Scaling stiffness energy

In order to study the sensitivity to boundary conditions we consider blocks of $A(L+1)$ Ising spins. The parameter A is the transverse area of the sample and L is its length in the direction in which differing boundary conditions are applied. For 2D samples considered here we set $A = L$.

In the transverse direction, periodic boundary conditions are applied. In the longitudinal direction, each of the spins in the first and last columns are fixed randomly in one of the $(2S + 1)$ states. The domain wall is created by turning the spin states on one boundary upside down with the spins on the other boundary held fixed. The difference in the ground-state energies is denoted by ΔE. Since it can be either positive or negative, we define

$$\delta E(L) = \langle |\Delta E| \rangle_c \tag{3}$$

as the characteristic measure of the influence of neighbouring blocks on the finite block under study. Here, $\langle \cdots \rangle_c$ denotes the configurational average over samples.

Our calculations were done using the transfer-matrix method which was generalized from the $S = \frac{1}{2}$ case, described by Morgenstern and Binder (1980), to higher values of S. Our analysis extends the studies of the $S = \frac{1}{2}$ Ising systems done by

Bray and Moore (1984, 1987) and by Cieplak and Banavar (1990) to larger values of L and then to $S = 1, \frac{3}{2}, 2, \frac{5}{2}$, and 3.

For each system we took 10 000 samples into account in the configurational average. Figure 2 shows the plot of δE versns L on the log–log plane for the GSG in the absence of anisotropy term Δ in the Hamiltonian. The exponent y is given by the slope of the line in the figure. The largest linear system sizes L studied for $S = \frac{1}{2}$, 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, and 3 were 16, 10, 8, 8, 7, 6, and 5 respectively. Unlike the ferromagnetic case, the 'amplitude' of δE is found to be essentially proportional to S^2. For a better comparison of the $S = \frac{1}{2}$ and $S = 1$ cases in figure 2, we rescaled the $S = \frac{1}{2}$ spins so that they took values ± 1, and not $\pm\frac{1}{2}$. The summary of the power law exponents obtained is shown in figure 1.

For the $S = \frac{1}{2}$ GSG in 2D Bray and Moore (1984) have obtained the exponent y of -0.29 and Cieplak and Banavar (1990) of -0.31. Both of these results were based on the linear system sizes L not exceeding 10. Our current value, based on significantly larger system sizes is

$$y = -0.33 \pm 0.03 . \tag{4}$$

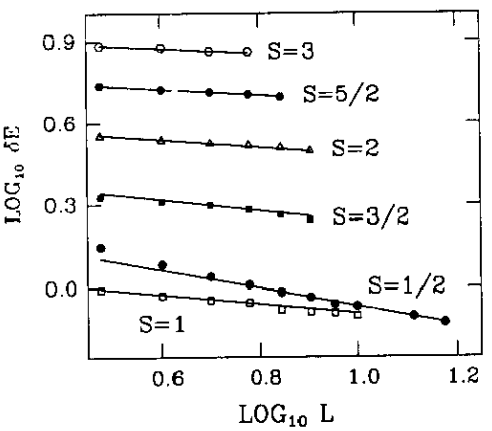

Figure 2. The L dependence of $\delta E(L)$ for two-dimensional systems with the Gaussian couplings and no uniaxial anisotropy. The corresponding spin quantum numbers are indicated. The full curves are fits to power laws. The spin takes values between $-S$ and S except for $S = \frac{1}{2}$ where the values are rescaled to ± 1.

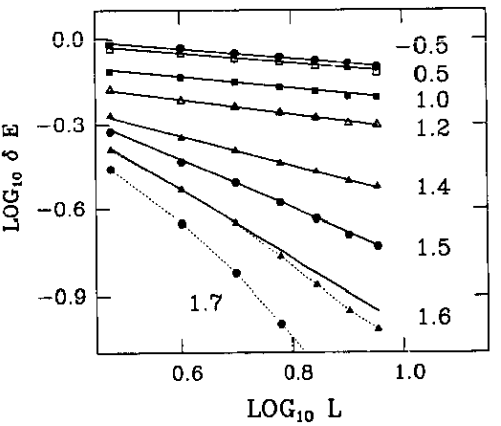

Figure 3. The L dependence of $\delta E(L)$ for two-dimensional $S = 1$ systems with the Gaussian couplings for the indicated values of Δ/J_0. The dotted curves connect the data points whereas the full curves are power-law fits.

When studying the bimodal case we introduced vacancies by setting 20% of the exchange couplings to zero. This was done to avoid the spurious odd-L–even-L effects on δE (Bray and Moore 1987, Cieplak and Banavar 1990). Extension of our studies beyond $L = 10$ suggests that the power law behaviour of δE also remains valid in this case and the exponent $y = -0.26 \pm 0.03$. Cieplak and Banavar (1990) have found an exponential behaviour of δE in the BSG case when A was fixed and only L was varied. This could have meant a paramagnetic behaviour at $T = 0$ but no deviations from the power law were observed when A and L varied together. Increasing the system size to 16 continues to indicate no deviations.

It is interesting to point out that we see no delta-function-like peak at zero defect energies in the distribution of ΔE even in the bimodal case. We also comment that in one-dimensional systems with Gaussian couplings $y = -1$ for any S that we studied.

We now focus on the $S = 1$ systems and consider the role of of the anisotropy in determining properties of a GSG. Figure 3 shows δE versus L on the log–log plane for selected values of Δ. For all negative and for small positive Δ the exponent y is equal to -0.17 ± 0.03. When $\Delta/J_0 = 0.8$ we get the first indications of a modified behaviour: the apparent exponent y becomes -0.19. This exponent continues to slide down with growing Δ; it is -0.20, -0.53, and -0.86 for Δ/J_0 equal to 1, 1.4, and 1.5 respectively. For still larger Δ the behaviour could be already identified as exponential, as indicated by the dotted lines which connect the data points. The straight lines are power law fits. The ground state of the system acquires a paramagnetic form since the state with zero spin projection is favoured too much. It is very likely that in the region where the apparent power law behaviour is observed but with a y different from -0.17, δE is actually described by

$$\delta E(L) \sim \mathrm{e}^{-L/\xi}/L^y \tag{5}$$

with a ξ exceeding our system size. If so, we conclude that the paramagnetic behaviour at $T = 0$ sets in around $\Delta/J_0 = 0.8$.

3. Migdal–Kadanoff analysis: $S = 1$

3.1. Recursion relations

The Migdal–Kadanoff approximation consists of a sequence of decimations and bond-moving operations on a D-dimensional Euclidean lattice. This MKRG scheme becomes exact for hierarchical lattices such as those shown in figure 4. In this scheme we first reduce b bonds in the series into one effective bond by decimating out spins in the middle. We then move the b^{D-1} effective bonds in parallel to form a hypercubic lattice with a lattice constant which is b times larger. As pointed out by Emery and Swendsen (1977a, 1977b) the MKRG scheme is ambiguous for $S > \frac{1}{2}$ since various combinations of couplings may be considered to constitute a 'bond' and one should decide on the best choice.

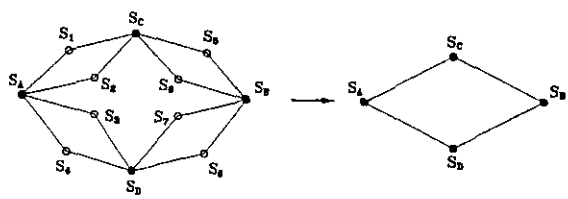

Figure 4. Rescaling transformation on a hierarchical lattice corresponding to $b = 2$ and $D = 2$.

To illustrate this point we consider a more general random Blume–Emery–Griffiths Hamiltonian (Blume *et al* 1971) with a biquadratic coupling

$$H = -\sum_{(ij)} J_{ij} S_i S_j - \sum_{(ij)} K_{ij} S_i{}^2 S_j{}^2 + \sum_i \Delta_i S_i{}^2 \tag{6}$$

where the spin S_i takes on values 0, ± 1, and we consider a uniform Δ. Since $S_i^4 = S_i^2$ for spin-1 Ising systems, the Hamiltonian (2) can be rewritten in the form

$$H = \sum_{(ij)} R(S_i, S_j) + \sum_i V(S_i^2) \tag{7}$$

where

$$R(S_i, S_j) = J_{ij}(S_i - S_j)^2/2 + K_{ij}(S_i^2 - S_j^2)^2/2 \tag{8}$$

$$V(S_i^2) = \Delta_i S_i^2 - \sum_{(j)_i} (J_{ij} + K_{ij}) S_i^2/2 \,. \tag{9}$$

Here $(j)_i$ denotes a summation over nearest neighbours of site i. The spin–spin coupling term in (8) is symmetric in the spin states and has the property that $R(S_i = S_j) = 0$. When doing the Migdal–Kadanoff renormalization one monitors flows of distributions of J_{ij}, K_{ij}, and Δ_i. The choice one has to make here is whether to monitor the coefficients in front of $S_i S_j$ and $S_i^2 S_j^2$ or in front of $(S_i - S_j)^2$ and $(S_i^2 - S_j^2)^2$. Emery and Swendsen have found that the latter choice, which requires rearrangement of the Hamiltonian from form (6) to form (7) after each rescaling, is the correct one for uniform ferromagnetic couplings. The reason is that this MKRG scheme becomes accurate both at high temperatures where the coupling itself is weak and at low temperatures where the coupling is strong but in the dominant configurations one has $S_i = S_j$. Thus in both limits one works with small Rs of (8).

We adopt the Emery–Swendsen prescription to the spin-glass systems and consider spins located on sites of hierarchical lattices. Since we deal with random systems the transformation from (6) to (7) has to be made locally for each bond. Another technical point is how one should split the anisotropy term $V(S_i^2)$ into bonds to get a reasonable account of this term. We take the prescription of Griffiths and Kaufman (1982), used to describe effects of a magnetic field, and adopt the following approximation. Consider a fragment of a $D = 2$, $b = 2$ hierarchical lattice as shown in figure 4. We notice that there are two cathegories of sites. The decimated sites, like those with spins S_1 and S_2 shown in figure 4, have 2 neighbours. On the other hand, the sites with spins S_A and S_C have, in general, 2^D neighbours since one combines b^{D-1} effective couplings on two sides of such spins. It is thus natural to postulate that the V-terms on decimated spins come with a weight of $\frac{1}{2}$ and on the undecimated sites with the weight of $(1/2)^D$.

After one rescaling step the coupling between undecimated sites, like A and C in figure 4, become J'_{AC}, K'_{AC}. Similarly, the renormalized anisotropy constants become Δ'_A and Δ'_C respectively. Then J'_{AC} and K'_{AC} are obtained by combining renormalized bonds from 2^{D-1} parallel segments in the block AC. However, to obtain Δ'_A and Δ'_C one has to combine 2^D contributions coming from decimations in the

blocks AC, AD and in the blocks AC and CB respectively. Thus to get a complete set of the recursion relations it is neccesary to carry out decimation in three blocks AC, AD and CB.

We now describe the decimation part of the rescaling procedure. Consider a segment AiC in the block AC. The index i takes 2^{D-1} values. After the Emery–Swendsen transformation the Hamiltonian H_{AiC} for the segment AiC becomes

$$H_{AiC} = R(S_A, S_i) + R(S_i, S_C) + V(S_A^2) + V(S_i^2) + V(S_C^2) \tag{10}$$

$$R(S_A, S_i) = J_{Ai}(S_A - S_i)^2/2 + K_{Ai}(S_A^2 - S_i^2)^2/2 \tag{11}$$

$$R(S_i, S_C) = J_{iC}(S_i - S_C)^2/2 + K_{iC}(S_i^2 - S_C^2)^2/2 \tag{12}$$

$$V(S_A^2) = \delta_A S_A^2 \qquad V(S_i^2) = \delta_i S_i^2 \qquad V(S_C^2) = \delta_C S_C^2 \tag{13}$$

where

$$\delta_A = \Delta_A/2^D - (J_{Ai} + K_{Ai})/2 \tag{14}$$

$$\delta_C = \Delta_C/2^D - (J_{iC} + K_{iC})/2 \tag{15}$$

$$\delta_i = \Delta_i - (J_{Ai} + K_{Ai} + J_{iC} + K_{iC})/2\,. \tag{16}$$

Then the decimation procedure reads

$$\exp(-H'_{AC}(i) + \text{constant}) = \mathrm{Tr}_{S_i}[\exp(-H_{AiC})] \tag{17}$$

$$H'_{AC}(i) = J'_{AC}(i)(S_A - S_C)^2/2 + K'_{AC}(i)(S_A^2 - S_C^2)^2/2 + \Delta'_A(C,i)S_A^2 + \Delta'_C(A,i)S_C^2 \tag{18}$$

where the trace Tr_{S_i} is taken over the states of spin S_i. The index i in $H'_{AC}(i)$, $J'_{AC}(i)$ and $K'_{AC}(i)$ merely indicates that these quantities are obtained after decimation over spin S_i; $\Delta'_A(C,i)$ denotes a contribution to the anisotropy on site A due to decimation over S_i between sites A and C (similarly for $\Delta'_C(A,i)$). The expressions for the couplings obtained as a result of decimation are written in the appendix. After decimation we obtain the recusion relations in the form

$$\begin{aligned}
J'_{AC} &= \sum_{i=1}^{2^{D-1}} J'_{AC}(i) \\
K'_{AC} &= \sum_{i=1}^{2^{D-1}} K'_{AC}(i) \\
\Delta'_A &= \sum_{i=1}^{2^{D-1}} \Delta'_A(C,i) + \sum_{j=2^{D-1}+1}^{2^D} \Delta'_A(D,j) \\
\Delta'_C &= \sum_{i=1}^{2^{D-1}} \Delta'_C(A,i) + \sum_{k=2^D+1}^{3.2^{D-1}} \Delta'_C(B,k)\,.
\end{aligned} \tag{19}$$

The symbols $\Delta'_{\mathrm{A}}(\mathrm{D},j)$ and $\Delta'_{\mathrm{C}}(\mathrm{B},k)$ indicate contributions to the renormalized anisotropy at sites A and C due to decimation in the blocks AD and CB. The expression of $\Delta'_{\mathrm{A}}(\mathrm{D},j)$ and $\Delta'_{\mathrm{C}}(\mathrm{B},k)$ may be obtained from $\Delta'_{\mathrm{A}}(\mathrm{C},i)$ by making the replacements $\mathrm{C} \Rightarrow \mathrm{D}$, $i \Rightarrow j$ and $\mathrm{C} \Rightarrow \mathrm{B}$, $\mathrm{A} \Rightarrow \mathrm{C}$ and $i \Rightarrow k$ respectively.

In a numerical realization of the MKRG scheme we start by generating a pool of typically $N_b = 100\,000$ exchange couplings J_{ij}. On the microscopic level, we put $K_{ij} = 0$ and take $\Delta_i = \Delta$ but in general we also have a pool of N_b biquadratic couplings and of N_b anisotropy constants. We then randomly draw the couplings and anisotropies from the three pools and combine them according to the recursion relations (19), (A1)–(A9). The new couplings and the anisotropy constants are collected into the pools of rescaled couplings. The procedure is continued until the pools gather N_b elements.

At each length scale we characterize the distributions of J_{ij}, K_{ij} and Δ_i by the means J_{av}, K_{av}, Δ_{av} and their dispersions σ_J, σ_K and σ_Δ, e.g.

$$J_{\mathrm{av}} = \langle J_{ij} \rangle \qquad \sigma_J = (\langle J_{ij}^2 \rangle - J_{\mathrm{av}}^2)^{1/2} \tag{20}$$

where $\langle \cdots \rangle$ denotes an average over configurations of the couplings in the pools. All of these quantities, as well as $k_{\mathrm{B}}T$, will be measured here in units of J_0.

3.2. The phase diagram on the T–Δ plane

The phase diagram may be obtained by investigating the scaling properties of the mean and dispersion of the couplings as a function of the system size $L = 2^n$, where n is the number of scaling steps, for various values of T and Δ. For both Gaussian and bimodal couplings J_{ij}, $J_{\mathrm{av}} = 0$. On the other hand, σ_J either grows or decreases depending on D, T, and Δ. At any non-zero T the power-law growth in σ_J indicates the spin-glass behaviour and the exponential decrease points to the paramagnetism. The behaviour of σ_K and σ_Δ is similar to that of σ_J whereas the behaviour of K_{av} and Δ_{av} are different. When σ_J increases K_{av} and Δ_{av} decrease monotonically. On the other hand, when σ_J decreases the mean K_{av} goes to zero but Δ_{av} approaches a fixed negative value in the iteration process.

For $D = 2$ and $S = 1$ there is no finite $T = 0$ transition for any Δ, in agreement with the transfer matrix calculations. For bimodal couplings J_{ij} we have paramagnetic phase at $T = 0$ for abitrary Δ. The 2D system with Gaussian couplings is paramagnetic at $T = 0$ for $\Delta > 0$ and becomes critical at $T = 0$ for $\Delta \leqslant 0$.

The phase diagram for GSG in $D = 3$ is shown in figure 5. A similar phase diagram is obtained for BSG. The paramagnet–spin-glass transition line corresponds to the scale invariance of the dispersion σ_J. Our calculations show that σ_K, σ_Δ, K_{av} and Δ_{av} also become scale-invariant on this transition line. At $\Delta = 0$ the critical temperature T_{c} is somewhat lower for GSG ($T_{\mathrm{c}} = 0.28J_0$) than for BSG ($T_{\mathrm{c}} = 0.35J_0$). These values are approximatly three times lower than those for the $S = \pm 1$ case (our calculations give $T_{\mathrm{c}} = 0.85J_0$ for GSG and $1.13J_0$ for BSG) obtained by a similar MKRG scheme. The critical values of Δ, Δ_{c}, above which the system becomes paramagnetic at $T = 0$ are $0.28J_0$ for GSG and $0.38J_0$ for BSG.

According to the mean-field theory (Ghatak and Sherrington 1977, Lage and de Almeida 1982, Mottishaw and Sherrington 1985), the spin-glass–paramagnet transition is generally second order but in a certain range $\Delta_{\mathrm{C1}} < \Delta < \Delta_{\mathrm{C2}}$ it becomes first order. Furthermore, the phase diagram was predicted to have a toppology that would allow a re-entrant behaviour in the same range of Δ. As seen in figure 5 our MKRG

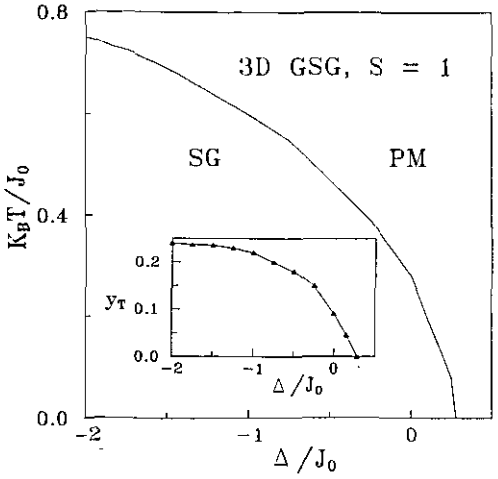

Figure 5. The temperature–anisotropy phase diagram for the $S = 1$ three-dimensional spin glass with Gaussian couplings is obtained by the Migdal–Kadanoff method. The critical value of Δ/J_0 at $T = 0$ is equal to 0.28. The inset shows the dependence of the thermal exponent y_T on Δ.

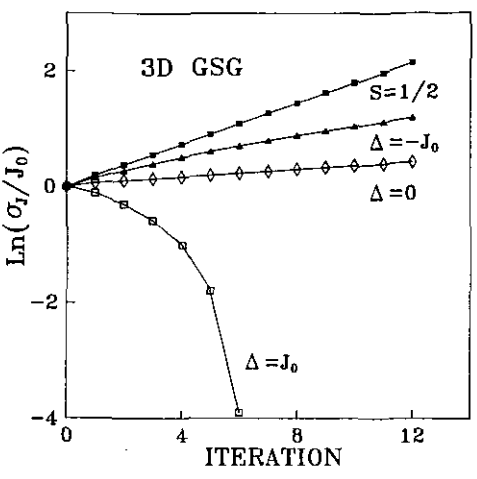

Figure 6. The dependence of the dispersion of the effective exchange couplings on the number of iterations for a three-dimensional $S = 1$ system with Gaussian couplings. The data points refer to $T = 0$. Results for $S = \frac{1}{2}$ are shown for comparison.

scheme does not lead to any re-entrancy which probably means that this feature is a mean-field phenomenon. This is confirmed by a recent determination of the T–Δ phase diagram (Hassan *et al* 1992) by the local mean-field method (Soukoulis *et al* 1982, 1983). The local mean-field methods yields no re-entrancy and suggests that the spin-glass–paramagnet transition is second order for any value of Δ.

The MKRG method used in the present paper also suggests that the phase transition is continuous. In order to see this we have calculated the thermal exponent y_T (Southern and Young, 1977). If this exponent were equal to the spatial dimensionality D the transition would be first order (Nienhuis and Nauenberg, 1975). We find, however, that y_T is always much smaller than 3 for $D = 3$. The exponent y_T is obtained by following the flow of σ_J, σ_K, σ_Δ, K_{av} and Δ_{av} near the transition. The exponent y_T is found to depend on the anisotropy constant and this dependence is shown in figure 5. At the $\Delta = \Delta_c$, $y_T = 0$. For all values of Δ the thermal exponent is smaller than the corresponding $S = \frac{1}{2}$ MKRG value. For the $S = \frac{1}{2}$ case and $D = 3$ we have obtained $y_T = 0.32$ which agrees with previous result $y_T = 0.36$ of Southern and Young (1977). (These authors have used the MKRG scheme and assumed that the exchange distribution remains Gaussian through the rescalings.)

Finally, we report that if J_{ij} is uniform and Δ_i is random with zero mean the system behaves like a uniform system without any anisotropy.

3.3. The $T = 0$ scaling exponent y

We now focus on the details of the behaviour of σ_J and of the other scaling quantities at $T = 0$. We first consider results obtained for the 3D case. Figure 6 shows the flow of σ_J for the GSG for three selected values of Δ. For $\Delta = J_0$ we clearly get a paramagnetic behaviour. Below a critical Δ_c a power law holds. The exponent y,

however, shows a weak dependence on Δ. This is shown in figure 7 where the 2D results are also combined for a comparison. In the MKRG scheme the length scales studied can be almost arbitrary. Thus unlike what we have found by the transfer matrix method the Δ-dependence of y seems to be a fallacy of the method and not a manifestation of an apparent power law arising due to an insufficient system size.

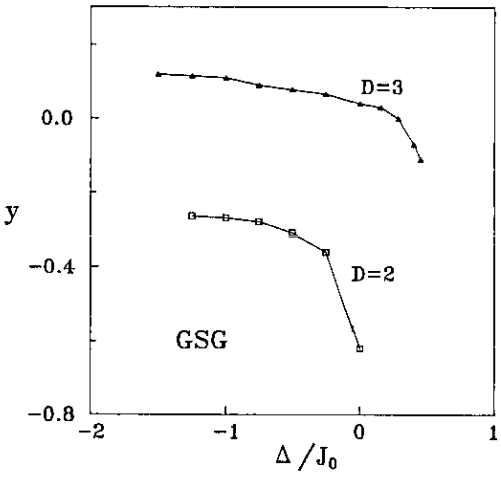

Figure 7. The dependence of the $T = 0$ scaling exponent y on Δ for two- and three-dimensional $S = 1$ systems. The MKRG method applied to $S = \frac{1}{2}$ spin glasses would yield $y = 0.26$ and -0.24 for the 3D and 2D GSG respectively.

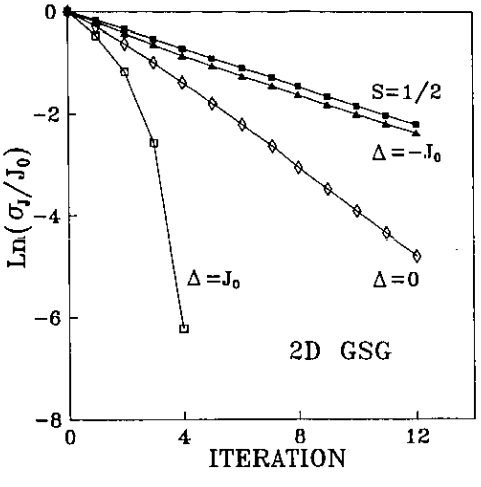

Figure 8. Same as in figure 6 but for two-dimensional $S = 1$ systems.

Both the 2D and 3D exponents are generally smaller than the corresponding $S = \frac{1}{2}$ MKRG values for GSG ($y = 0.26$ in 3D and $y = -0.24$ in 2D; see for example, Cieplak and Banavar (1990)). For $\Delta/J_0 = -1.0$ and 0.0, e.g. y is equal to 0.11 and 0.04 respectively. Thus the MKRG approach predicts that the exponent y depends on the spin quantum number S, which is in a qualitative agreement with the transfer matrix result. As long as $\Delta/J_0 \leqslant 0.15y$ for 3D, GSG is essentially the same as for the BSG system.

In 2D systems there is no finite T transition but in the GSG case there appears to be a $T = 0$ transition from a power law with a negative exponent to an exponential decay at $\Delta \leqslant 0$. This is shown in figure 7. Figure 8 shows the flow of σ_J for three selected values of Δ. In the BSG case, on the other hand, the MKRG method predicts the systems to be paramagnetic at any Δ, which is similar to the results obtained by the same method for the $S = \frac{1}{2}$ BSG system. This paramagnetism at $T = 0$ is likely to be an artifact of the approximations used (see Cieplak and Banavar 1990). For example, the transfer-matrix method, as one can see in section 2, gives $y = -0.17$ for 2D BSG.

3.4. Lower critical dimensionality

The procedure to find the LCD is similar to the one used by Caflisch and Banavar (1985) to study the $S = \frac{1}{2}$ spin glasses. We have to combine $d_b = 2^{D-1}$ of decimated

bonds, with D allowed to be non-integer, into one rescaled coupling. By trial and error we find d_b which yields scale invariance. For instance, for the $S = 1$ and $\Delta = 0$ it turns out that we are required to combine 3.78 such bonds at the LCD. Thus we combine three decimated bonds with a weight of 1 and take the fourth bond with a weight of 0.78. The LCD is found to be lower than 3 for $\Delta < \Delta_c$. For example, we find LCD of 2.78 and 2.94 for $\Delta = -J_0$ and 0 respectively. Above Δ_c the LCD is greater than 3. Note that for $\Delta = 0$ the LCD of the $S = 1$ system is greater than that of the $S = \frac{1}{2}$ system (of (2.55)) which was obtained by the same MKRG method (Caflish and Banavar, 1985). The transfer-matrix results for $D = 2$ suggest, however, that the LCD of the $S = 1$ model should actually be lower because the exponent y was larger. Thus the tendencies suggested by our MKRG scheme are opposite to those determined by the transfer-matrix method.

4. Concluding Remarks

Results on the frustrated $S = 1$ Ising systems obtained by the MKRG scheme with the use of the Emery–Swendsen trick are comparable in quality to those obtained for the $S = \frac{1}{2}$ glasses. The physics of the $S = 1$ spin glasses appears to be similar to that of the $S = \frac{1}{2}$ ones. There is no finite T ordering in two dimensions but there is an ordering in three dimensions. An extra degree of freedom, the single-ion anisotropy, can modify T_c and even bring it down to 0 but it seems that in the short-range systems there is no re-entrancy and no first-order transition predicted by the mean-field theory.

The transfer-matrix results, however, point to a new phenomenon: the $T = 0$ exponents of frustrated systems depend on the spin quantum number. Furthermore, it is possible that half-integer S spin glasses belong to a different branch of the exponents than the integer S systems. An experimental verification of these predictions would be welcome.

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Appendix

Expressions of $J'_{AC}(i)$, $K'_{AC}(i)$, $\Delta'_A(C,i)$ and $\Delta_C(A,i)$ for the $S = 1$ case are as follows:

$$J'_{AC}(i) = k_B T(\ln \Sigma_1 - \ln \Sigma_2)/2 \tag{A1}$$

$$K'_{AC}(i) = k_B T(\ln \Sigma_4 + \ln \Sigma_3 - \ln \Sigma_2 - \ln \Sigma_5) \tag{A2}$$

$$\Delta'_A(C,i) = k_B T(\ln \Sigma_4 + \ln \Sigma_5 - \ln \Sigma_1 - \ln \Sigma_2)/2 + [J'_{AC}(i) + K'_{AC}(i)]/2 \tag{A3}$$

$$\Delta'_C(A,i) = k_B T(\ln \Sigma_4 + \ln \Sigma_2 - \ln \Sigma_5 - \ln \Sigma_1)/2[J'_{AC}(i) + K'_{AC}(i)]/2 \tag{A4}$$

where

$$\Sigma_1 = \exp[-(\delta_A + \delta_C + \delta_i)/k_B T] + \exp\{-[(J_{Ai} + J_{iC} + K_{Ai} + K_{iC})/2 + \delta_A + \delta_C]/k_B T\} + \exp[-(2J_{Ai} + 2J_{iC} + \delta_A + \delta_i + \delta_C)/k_B T] \quad (A5)$$

$$\Sigma_2 = \exp\{-[(J_{iC} + K_{iC})/2 + \delta_A + \delta_i]/k_B T)\} + \exp\{-[(J_{Ai} + K_{Ai})/2 + \delta_A]/k_B T\} + \exp[-(2J_{Ai} + (J_{iC} + K_{iC})/2 + \delta_A + \delta_i)/k_B T] \quad (A6)$$

$$\Sigma_3 = \exp[-(2J_{iC} + \delta_A + \delta_i + \delta_C)/k_B T] + \exp\{-[(J_{Ai} + J_{iC} + K_{Ai} + K_{iC})/2 + \delta_A + \delta_C]/k_B T\} + \exp[-(2J_{Ai} + \delta_A + \delta_i + \delta_C)/k_B T] \quad (A7)$$

$$\Sigma_4 = 1 + \exp[-(J_{Ai} + J_{iC} + K_{Ai} + K_{iC})/k_B T] \quad (A8)$$

$$\Sigma_5 = \exp\{-[(J_{Ai} + K_{Ai})/2 + \delta_i + \delta_C]/k_B T\} + \exp\{-[(J_{iC} + K_{iC})/2 + \delta_C]/k_B T\} + \exp\{-[(J_{Ai} + K_{Ai})/2 + 2J_{iC} + \delta_i + \delta_C]/k_B T\}. \quad (A9)$$

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